Math 26L.04 Spring 2002
Extra Review Problems for Test #3

1. Look over Assignment #6, Assignment #7, Assignment #8, Quiz #6, Quiz #7, Quiz #8, and the extra problems solved in the class notes.

2. Show that $\displaystyle\int \frac{x^2-1}{(x^2+x+1)^2}\,dx = \frac{x^2+1}{x^2+x+1} + C.$

(Hint: Substitution and parts will not work. What does this statement ***mean****?)*

3. Evaluate the following integrals.

(a) $\displaystyle\int \tan x\,dx$

(b) $\displaystyle\int_1^3 x^2 \ln x\,dx$

(c) $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx$

4. Solve the initial value problem

$$\frac{x}{\ln x}\frac{dy}{dx} - \frac{1}{y\sin y} = 0,\; y(1) = \pi.$$

5. Consider $\displaystyle\int_0^4 \frac{1}{1+a^x}\,dx$ where $a \geq 1$ is a constant.

(a) Use a left hand Riemann sum with 4 subintervals to approximate the value of this integral. *(Naturally, your answer will involve the constant a.)*

(b) Use a right hand Riemann sum with 4 subintervals to approximate the value of this integral. *(Naturally, your answer will involve the constant a.)*

(c) How do the left hand and right hand Riemann sums compare to the true value of $\displaystyle\int_0^4 \frac{1}{1+a^x}\,dx$? Why?

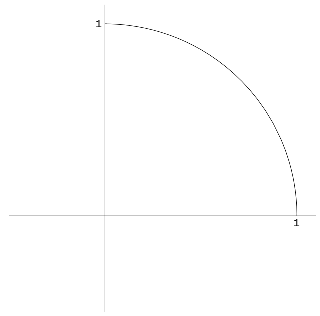
(d) If $a = 2$, use a left hand Riemann sum with 100 subintervals to approximate the value of this integral.

(e) If $a = 2$, use a right hand Riemann sum with 100 subintervals to approximate the value of this integral.

6.

(a) State the arc length formula for the length of the arc of the curve $f(x)$ between $x = a$ and $x = b$.

Consider the arc of the unit circle $x^2 + y^2 = 1$ in the first quadrant as shown below.



(b) Without doing any calculus, write down the length of the arc of the unit circle in the first quadrant.

(c) Use the arc length formula to determine the length of the arc of the unit circle in the first quadrant, and thus verify your answer from **(b)**.

7. In the study of probability, the *Gamma function* is often very useful.

The *Gamma function*, $\gamma(x)$, with unknown p is defined for $x \geq 0$ by

$$\gamma(x) = Gx^{p-1}e^{-x}$$

where G is called the *Gamma constant*.

The *Gamma constant* is chosen so that the value of the definite integral of $\gamma(x)$ from $x = 0$ to $x = \infty$ is 1.

Suppose that $p = 3$. Find the *Gamma constant*.

(That is, find the value of G so that $\int_0^\infty Gx^{p-1}e^{-x}\,dx = 1$ when $p = 3$.)

8. Suppose that $f(x)$ and $g(x)$ satisfy $f(0) = 5$, $f(1) = 3$, $f(2) = 2$, and $g(0) = 2$, $g(1) = 0$, $g(2) = 3$, respectively.

(a) Evaluate $\int_0^1 f'(g(x))g'(x)\,dx$.

(b) Evaluate $\int_0^1 f(x)g'(x)\,dx + \int_0^1 f'(x)g(x)\,dx$.