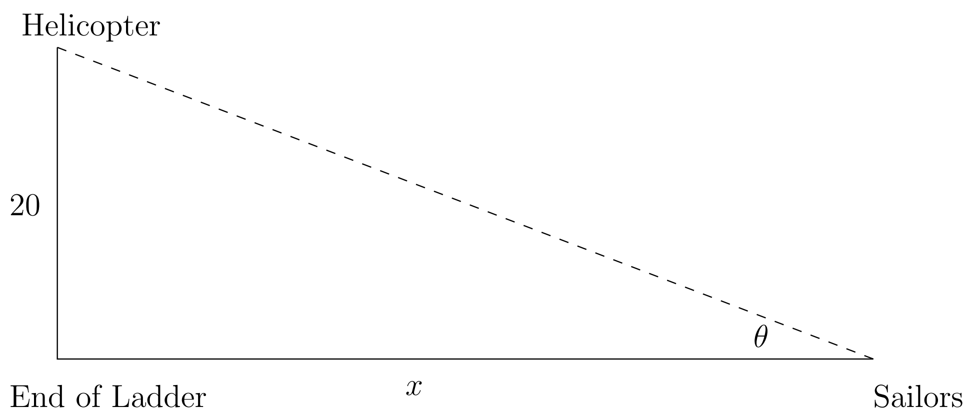
Math 026L.04 Spring 2002
January 16, 2002

Example: A group of sailors are shipwrecked on an island and a helicopter is coming to rescue them.

The helicopter has a hanging 20 ft. ladder and is hovering at a constant 20 ft. above the sea. It is approaching the shipwrecked sailors at 10 ft./sec.

How fast is the angle of elevation from the sailors to the helicopter changing when the ladder is 200 ft. away?

Solution: Let θ be the angle of elevation and let x be the distance from the end of the ladder to the sailors.



IMPORTANT: SINCE BOTH THE ANGLE θ AND THE DISTANCE x ARE CHANGING AS THE HELICOPTER GETS CLOSER, THEY ARE BOTH DEPENDENT ON TIME.

That is, θ is really a function of t, and x is also a function of t. In other words, $\theta = \theta(t)$ and $x = x(t)$.

From the diagram, we see that x and θ are related by tangent.

$$\therefore \tan(\theta) = \frac{20}{x}.$$

Taking derivatives with respect to t we get

$$\frac{d}{dt}\tan(\theta) = \frac{d}{dt}\frac{20}{x}.$$

By the chain rule this gives,

$$\frac{1}{\cos^2(\theta)} \cdot \frac{d\theta}{dt} = \frac{-20}{x^2} \cdot \frac{dx}{dt}.$$

continued

Since the helicopter is *approaching* the sailors at 10 ft./sec., $\frac{dx}{dt} = -10$.

We are interested in $\frac{d\theta}{dt}$ when $x = 200$, so we find that when $x = 200$, $\cos(\theta) = \frac{200}{\sqrt{40400}} \approx .5445$.

Solving for $\frac{d\theta}{dt}$ gives

$$\frac{d\theta}{dt} = \cos^2(\theta) \cdot \frac{-20}{x^2} \cdot \frac{dx}{dt} = \left(\frac{200}{\sqrt{40400}}\right)^2 \cdot \frac{-20}{200^2} \cdot -10 = 0.00495$$

Thus when the helicopter is 200 ft. from the sailors, the angle of elevation is changing at 0.00495 radians/sec.