Math 025L.01 Fall 1999

Test #3

1. (*12 points*) Suppose that $f(x) = \ln(x^2 + e^x)$, $g(x) = \sqrt{x}$, and $h(x) = 2^x + x^2$.

a. (*6 pts*) Compute $k'(x)$ if $k(x) = f(g(x)) + f(x)h(x)$.

b. (*6 pts*) Compute $k'(x)$ if $k(x) = \dfrac{f(x)}{g(x)} - h(g(x))$.

2. (*15 points*) A recent Math 31L test asked the students to compute $\dfrac{d}{dx}x^x$. After the test, I met two of these Math 31L students sitting in the help room.

a. (*5 pts*) The first student said that he had "gotten the right answer" and that it had to be $x \cdot x^{x-1}$. What has he done and why is his "answer" wrong?

b. (*5 pts*) The second student said that she had "gotten the right answer" and that it had to be $\ln(x) \cdot x^x$. What has she done and why is her "answer" wrong?

c. (*5 pts*) What does $\dfrac{d}{dx}x^x$ actually equal?

3. (*8 points*) Suppose that both $f(x)$ and $g(x)$ are differentiable functions. Use the definition of derivative to prove directly that $(f(x) - g(x))' = f'(x) - g'(x)$. Be sure to clearly state the properties of limits that you use.

4. (*10 points*)

a. (*5 pts*) A spherical snowball is melting in such a way that its surface area is decreasing at a rate of 1 cm^2/min. At what rate is the radius decreasing when the radius is 5 cm?

b. (*5 pts*) Suppose instead that the snowball is melting so that its volume is decreasing at a rate of 1 cm^3/min. Find the rate at which the radius is decreasing when the radius is 5 cm.

5. (*11 points*) The curve defined by the equation $y^2 = 5x^4 - x^2$ is called the **kampyle of Eudoxus**.

a. (*4 pts*) Find $\dfrac{dy}{dx}$.

b. (*2 pts*) Find the equation of the tangent line to the kampyle of Eudoxus at the point $(1, 2)$.

c. (*5 pts*) At what points does the kampyle of Eudoxus have a vertical tangent?

6. (*12 points*) The function f has the following properties:

- $f(-1) = 0$, $f(0) = 1$, $f(1) = 2$
- $f'(x) = \dfrac{2(1 - x^2)}{(1 + x^2)^2}$

a. (*6 pts*) Find the intervals on which f is increasing and decreasing, and any local maximum or minimum values of f.

b. (*6 pts*) Find the intervals on which the graph of f is concave up and concave down. At what values of x does the graph of f have an inflection point?

7. (*10 points*) While at a clinic in Roxboro, North Carolina, a patient was given a dose of Xylopain, after which he lost consciousness. He was rushed to Duke Hospital where the level of the drug was checked hourly. The readings are recorded in the following table:

hours elapsed since dose of Xylopain	2	3	4	5	6	7	8
concentration of Xylopain in mg/ml	6	4.25	3	2.12	1.5	1.06	.75

a. (*7 pts*) Find a function, $C(t)$, that approximates the level of Xylopain in the patient's blood t hours after the drug was given to the patient.

b. (*3 pts*) Using this function, find the concentration of Xypolain in the patient's blood immediately after the drug was given to him. Has this concentration exceeded the maximum allowable concentration of Xylopain which is 10 *milligrams per milliliter*?

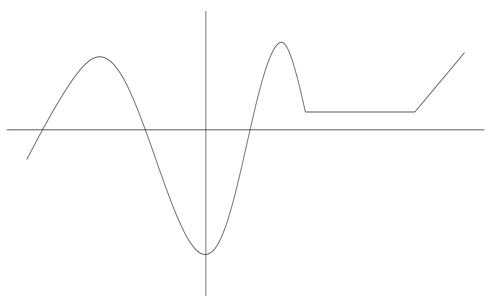
8. (*10 points*) Two variable resitors, R and S, are connected in parallel so that their combined resitance C is given by

$$\frac{1}{C} = \frac{1}{R} + \frac{1}{S}.$$

At an instant when $R = 250$ ohms and $S = 1,000$ ohms, R is increasing at a rate of 100 ohms/minute. How fast must S be changing at that moment if C is increasing at a rate of 10 ohms/minute?

9. (*6 points*) Use an appropriate linear approximation to estimate the value of $e^{0.1}$.

10. (*6 points*) Below is the graph of the function $f(x)$.



Identify all the critical points of f and clearly label them on the graph.

Bonus Question (*2 bonus points*)

Give an informal argument using a linear approximation to explain why the "very important theorem" must be true. Remember that the very important theorem states that if $f(x)$ is differentiable at $x = a$, then it is continuous at $x = a$.

Survey Question (*1 bonus point*)

What did you think of this test? Was it what you were expecting?