Math 103.01 Summer 2001
Test #2

Name: ______________________________

Read all of the following information before starting the test:

- Be sure that this test has **10** pages including this cover.
- There are **8** problems on this test worth a total of **100** points.
- The last page is for your scrap work and may be detached from the test booklet.
- Calculators are permitted, but no other aids are allowed.
- Show all work neatly and in order, and clearly indicate your final answers.
- Answers must be justified whenever possible in order to earn full credit. ***No credit will be given for unsupported answers, even if your final answer is correct.***
- Please keep your written answers succinct. Points will be deducted for incoherent, incorrect and/or irrelevant statements.
- Good luck!

Problem	1	2	3	4	5	6	7	8	Total
Score									

1. (*15 points*) Find all critical points of the function $f(x, y) = e^{-y}(x^2 - y^2)$ and classify each critical point as either a local maximum, a local minimum, or a saddle point.

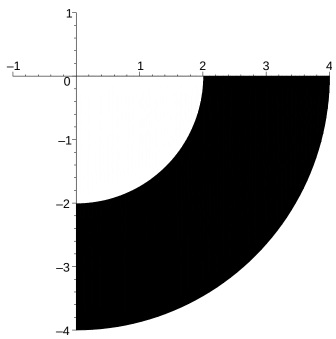
2. (*15 points*) Find the absolute maximum value and the absolute minimum value of $f(x, y) = xy$ subject to the constraint $8x^2 + y^2 \leq 48$.

3. (*12 points*) Evaluate

$$\int_0^1 \int_{y^2}^1 y \sin(x^2)\ dxdy.$$

4. (*12 points*) Let R be that part of the annulus which lies in the fourth quadrant formed by removing a semicircular disk of radius 2 from a semicircular disk of radius 4. Compute

$$\int\int_R \frac{x}{\sqrt{x^2+y^2}}\, dA.$$



5. (*12 points*) Compute

$$\int\int\int_T 1 - y^2 \, dV$$

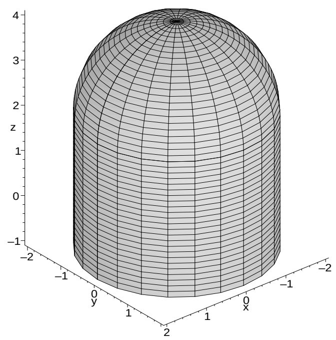
where T is the tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,2,0)$, $(0,0,3)$.

6. (*12 points*) Suppose that T is the region in the first octant which lies between the spheres $x^2 + y^2 + z^2 = 2$ and $x^2 + y^2 + z^2 = 4$. That is, T is defined by the inequalities $x \geq 0$, $y \geq 0$, $z \geq 0$, and $2 \leq x^2 + y^2 + z^2 \leq 4$. Evaluate

$$\int\int\int_T \frac{z}{x^2 + y^2}\, dV.$$

7. (*12 points*) Let T be the solid which lies within the cylinder $x^2 + y^2 = 4$, above the plane $z = -1$, and below the sphere $x^2 + y^2 + (z-1)^2 = 4$. Compute

$$\iiint_T z \, dV.$$



8. (*10 points*) As you may know, the area of the unit disk $x^2 + y^2 = 1$ in $\mathbb{R}^2$ is π and the volume of the unit sphere $x^2 + y^2 + z^2 = 1$ in $\mathbb{R}^3$ is $\frac{4}{3}\pi$.

There is a natural generalization to 4 dimensions. The surface $x^2 + y^2 + z^2 + w^2 = 1$ is the called the unit hypersphere in $\mathbb{R}^4$.

Write down, but do not evaluate, a quadruple integral to express the hypervolume of the unit hypersphere in $\mathbb{R}^4$.

This quadruple integral may be computed by a generalization of spherical coordinates to four dimensions. In fact, the hypervolume of the unit hypersphere in $\mathbb{R}^4$ is $\frac{\pi^2}{2}$.

Scrap Page

(*You may carefully remove this page from the test booklet.*)