Math 103.01 Summer 2001
Assignment #1
Due: Thursday, May 24, 2001

You must work through all problems on your own. You may consult any reference materials but do not discuss these problems with anyone else in the class. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit.

1. A ship at position $(1, 0)$ on a nautical chart (with north in the positive y direction) sights a rock at position $(2, 4)$.

a. What is the vector joining the ship to the rock?

b. What angle θ does this vector make with due north? (This is called the *bearing* of the rock from the ship.)

Suppose that the ship is pointing due north and traveling at a speed of 4 knots relative to the water. There is a current flowing east at 1 knot; the units on the chart are nautical miles; 1 knot = 1 nautical mile per hour.

c. If there were no current, what vector $\mathbf{u}$ would represent the velocity of the ship relative to the sea bottom?

d. If the ship were just drifting with the current, what vector $\mathbf{v}$ would represent its velocity relative to the sea bottom?

e. What vector $\mathbf{w}$ would represent the total velocity of the ship?

f. Where would the ship be after 1 hour?

g. Should the captain chage course?

h. What if the rock were an iceberg?

2. Find an equation of the plane with x-intercept a, y-intercept b, and z-intercept c.

3. Find parametric equations for the line through the point $(0, 1, 2)$ that is parallel to the plane $x + y + z = 2$ and perpendicular to the line $x = 1 + t$, $y = 1 - t$, and $z = 2t$.

4. Suppose that L_1 is the line with parametric equations $x = t, y = -6t + c, z = 2t - 8$ and that L_2 is the line with parametric equations $x = 3t + 1, y = 2t, z = 0$. Find the value of c for which the lines L_1 and L_2 intersect.

5. Suppose that $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in $\mathbb{R}^3$. Suppose further that α and β are real numbers. Prove the following:

a. $\alpha(\mathbf{u} + \mathbf{v}) = \alpha\mathbf{u} + \alpha\mathbf{v}$

b. $(\alpha + \beta)\mathbf{u} = \alpha\mathbf{u} + \beta\mathbf{u}$

c. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$

d. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$

e. $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \cdot \mathbf{w})\mathbf{v} + (\mathbf{u} \cdot \mathbf{v})\mathbf{w}$

f. $|\mathbf{u} \times \mathbf{v}|^2 = |\mathbf{u}|^2|\mathbf{v}|^2 - (\mathbf{u} \cdot \mathbf{v})^2$