Math 171 Prelim #1 – February 26, 2004

Thurston Hall, Room 203: Section 01 (Bendikov) & Section 02 (Kozdron)
Thurston Hall, Room 205: Section 03 (Hwang) & Section 04 (Hwang)

*You have **90** minutes to complete this exam. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit.*

Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct. *Points will be deducted for incoherent, incorrect, and/or irrelevant statements. Calculators are permitted, but no other aids are allowed. A table of normal probabilities will be provided.*

You are allowed to use standard notation. However, any new notation or abbreviations that you introduce must be clearly defined.

*This examination consists of **7** problems and is worth **100** total points. Each sub-part is worth 5 points, except that 7(a) is worth 7 points, and 7(b) is worth 8 points.*

All work must be completed in the examination booklets provided.

Good luck!

1. (*15 points*) In a recent survey of Cornell graduates, it was found that the monthly income of male graduates has a mean of $5,000 and a standard deviation of $2,000. It was also found that the monthly income of female graduates has a mean of $5,000 and a standard deviation of $3,000. Suppose you randomly pick a female graduate and randomly pick a male graduate.

(a) Carefully define random variables and use them to answer the following questions. What is their total monthly income? What is the difference in their monthly incomes?

(b) Making whatever assumptions are necessary in order to arrive at a numerical answer, use (a) to calculate the mean of their total monthly income, and the mean of the difference in their monthly incomes. If you require additional assumptions, what are they?

(c) Making whatever assumptions are necessary in order to arrive at a numerical answer, use (a) to calculate the standard deviation of their total monthly income, and the standard deviation of the difference in their monthly incomes. If you require additional assumptions, what are they?

2. (*10 points*) At least one-half of a ValueJet airplane's engines are required to function in order for the airplane to operate. Suppose that each ValueJet engine independently functions with probability 0.65. Decide which ValueJet airplane is safer to operate by answering the following two questions.

(a) What is the probability that a ValueJet two-engine plane is operational?

(b) What is the probability that a ValueJet four-engine plane is operational?

3. (*20 points*) Consider the following experiment. You roll a fair six-sided die, three of whose faces are painted red, two of whose faces are painted green, and one of whose faces is painted yellow. If you roll a yellow face, then you flip two fair coins. If you roll a green face, then you flip one fair coin. However, if you roll a red face, then you do not flip any coins.

(a) List all elements in the sample space, S. Clearly define any abbreviations you use.

(b) Let A be the event that you do not flip any coins. Compute $P(A)$.

(c) Let B be the event that you flip exactly two heads. Compute $P(B)$.

(d) Are the events A and B independent? Why or why not?

4. (*10 points*) You ask your neighbor to water a sickly plant while you are on vacation. Without water, the plant will die with probability 0.8. With water, the plant will die with probability 0.15. You are 90% certain that your neighbor will remember to water the plant.

(a) What is the probability that the plant will be alive when you return?

(b) If the plant is dead when you return, what is the probability that your neighbor forgot to water it?

5. (*15 points*) Data from the National Oceanic and Atmospheric Administration indicate that the yearly precipitation in Ithaca is a normal random variable with a mean of 12 inches, and a standard deviation of 3 inches. Assume that the precipitation for the next two years are independent.

(a) Find the probability that next year's precipitation is between 9 inches and 18 inches.

(b) What must next year's precipitation be so that there is an 80% chance of receiving at least that much precipitation?

(c) What are the mean and the standard deviation of the total precipitation for the next two years?

6. (*15 points*) Assume that a person independently tosses two different biased coins. The probability of getting a head on the first coin is 0.6, while the probability of getting a head on the second coin is 0.3.

(a) What is the expectation of the total number of heads in the two tosses?

(b) What is the variance of the total number of heads in the two tosees?

(c) What is the probability of getting exactly one head in the two tosses?

7. (*15 points*) Harley-Davidson motorcycles make up 14% of all the motorcycles registered in the United States. You plan to interview a simple random sample of 500 motorcycle owners.

(a) What is the approximate distribution of the proportion of your sample that own Harleys? Be sure to specify the mean and standard deviation explicitly.

(b) Is your sample likely to contain at least 15% who own Harleys? Answer this question by using a central limit theorem calculation.