Math 171 **Prelim 1** **Spring 2003**

1. Each year a company must send 3 officials to a meeting in China and 5 officials to a meeting in France. Airline ticket prices vary from time to time. Past experience has shown that tickets to China have a mean price of \$1000 with standard deviation \$150, while the mean airfare to France is \$500 with standard deviation \$100.

a) (6%) Define random variables and use them to express the total amount the company will have to spend to send these delegates to the two meetings.

b) (9%) Find the mean and standard deviation of this total cost.

c) (9%) Find the mean and standard deviation of the difference in price of a ticket to China and a ticket to France.

d) (4%) Do you need to make any extra assumptions in calculating these means? How about the standard deviation?

2. A psychology department at a university finds that the scores of its applicants on SAT math are approximately normal with mean 544 and standard deviation 103.

a) (8%) Find the approximate percentage of applicants having 544 or higher.

b) (8%) Find the approximate percentage of applicants having scores lower than 444.

c) (8%) An applicant's score is better than 90% of the applicants. What is her score?

3. It is known that 49% of new born babies are girls. A random sample of 800 babies have been chosen.

a) (4%) What is the approximate distribution of the sample proportion of girls with what mean and standard deviation?

b) (8%) What is the approximate probability that the sample proportion of girls is higher than 50%?

c) (8%) What is the approximate probability of the sample proportion of girls lying in (48%, 52%)?

4. (12%) Samuel Pepys wrote to Isac Newton: "What is more likely (a) at least one 6 in 6 rolls; (b) at least two 6's in 12 rolls? Calculate these two probabilities and answer Pepys' question.

5. Two couples are invited to go to dinner. The probability that couple A shows up is 0.9 and for couple B 0.8. Assume they show up independently.

a) (8%) What is the probability that both show up? What is the probability that nobody shows up?

b) (8%) Let X be the number of couples that will show up. What is $P(X = 1)$? What is the expectation and the standard deviation of X?

Math 171 first prelim Feb. 20th 2003

MATH 171 - PRELIM 1 - SOLUTIONS

1. a) C, F = ticket prices; A = total $

$A = 3C + 5F$

b) $E(A) = E(3C+5F) = E(3C) + E(5F) =$
$= 3E(C) + 5E(F) = 3(1000) + 5(500)$
$= \$5500$

$Var(A) = Var(3C) + Var(5F) =$
$= 3^2 Var(C) + 5^2 Var(F)$
$= 9 \cdot 150^2 + 25 \cdot 100^2$
$= 452500$

$SD(A) = \sqrt{452500} = \$672.68$

c) Let D = difference in price

$D = C - F$

$E(D) = E(C) - E(F) = \$500$

$Var(D) = Var(C) + Var(F) =$
$= 150^2 + 100^2 = 32500$

$SD(D) = \sqrt{32500} = \$180.28$

d) Means: No

SD's: Yes; ticket prices independent

e) No extra assumption needed for expectation

Independence assumption of X and Y is needed for standard deviation

2. Let X be the SAT of a randomly chosen applicant

a) $P(X > 544) = .5$ by symmetry of normal

b) $P(X < 444) = P\left(\frac{X-544}{103} < \frac{444-544}{103}\right)$
$= P(X < -0.97) = .166$

c) $.90 = P(X < q)$ where q is the answer
$= P\left(\frac{X-544}{103} < \frac{q-544}{103}\right) = P\left(Z < \frac{q-544}{103}\right)$

Since $P(Z < 1.28) = 0.9$ from Table A,

$\frac{q-544}{103} = 1.28$

$\Rightarrow q = 544 + (103)(1.28) = 675.8$

3 a) Let $\hat{p}$ be the sample proportion

$\hat{p} \overset{\text{appro.}}{\sim} N(p, \sqrt{pq/n})$
$= N\left(.49, \sqrt{\frac{(.49)(.51)}{200}} = 0.0177\right)$

b) $P(\hat{p} > .50) = P\left(\frac{\hat{p}-.49}{.0177} > \frac{.5-.49}{.0177}\right)$
$= P(Z > .5658) = 1 - .7157 = .285$

c) $P(.48 < \hat{p} < .52) = P\left(\frac{.48-.49}{.0177} < Z < \frac{.52-.49}{.0177}\right)$
$= P(-.56 < Z < 1.69)$
$= .9545 - .2877 = .6668$

4. Let X be the number of 6's

a) $P(X \geq 1) = 1 - P(X = 0)$

$X \sim B(6, \frac{1}{6})$

Hence $P(X=0) = \binom{6}{0}\left(\frac{5}{6}\right)^6 = .3349$

$\Rightarrow P(X \geq 1) = P(\text{at least one 6})$
$= 1 - .3349 = .6651$

b) $X \sim B(12, \frac{1}{6})$

$P(\text{at least 2 sixes}) = P(X \geq 2)$
$= 1 - P(X=0) - P(X=1)$
$= 1 - \binom{12}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{12} - \binom{12}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{11} = .6187$

$\Rightarrow$ Conclude a) is larger

5. (a) $P(\text{both show up}) = (.9)(.8) = .72$

$P(\text{none show up}) = (0.1)(0.2) = 0.02$

b)

X	0	1	2
Prob	0.02	0.26	0.72

$\Rightarrow P(X=1) = 1 - 0.02 - 0.72 = 0.26$

$E(X) = 0(0.02) + 1(0.26) + 2(0.72) = 1.7$

$Var(X) = E(X^2) - (1.7)^2 = 1^2(0.26) + 2^2(0.72) - (1.7)^2 = 0.25$

$SD(X) = 0.5$.