Make sure that this examination has 12 numbered pages

Cornell University
Final Examination
August 4, 2003

Mathematics 111
Calculus I

Closed Book Examination Time: 2 hours

Name: _____

Instructor: Michael Kozdron **Section:** 01

Read all of the following information before starting the exam.

*You have **2** hours to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit.* ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

Calculators are permitted, but do not replace exact expressions with decimal approximations. There are no other aids allowed.

*This test has **12** numbered pages with **14** questions totalling **200** points. Before you hand in your exam, make sure you have all of the pages.*

DO NOT WRITE BELOW THIS LINE

Page 1 _____	Page 6 _____	Page 11 _____
Page 2 _____	Page 7 _____	Page 12 _____
Page 3 _____	Page 8 _____	Page 13 _____
Page 4 _____	Page 9 _____	Page 14 _____
Page 5 _____	Page 10 _____	TOTAL _____

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TOTAL _____

1. (*15 points*) Compute the following. Do not simplify your answers.

(a) $\lim_{x \to 1} \dfrac{x^{16} - 1}{2x - 2x^{16}}$

(b) $\lim_{x \to \infty} \dfrac{x^{16} - 1}{2x - 2x^{16}}$

(c) $\lim_{x \to 0} e^{-x} \ln |x|$

2. (*15 points*) Compute the following. Do <u>not</u> simplify your answers.

(a) $\dfrac{d}{dx}\left(\dfrac{x^{16}-1}{2x-2x^{16}}\right)$

(b) $\dfrac{d}{dx}\arccos(e^x)$

(c) $\dfrac{d}{dx}\left(5^x \sin x\right)$

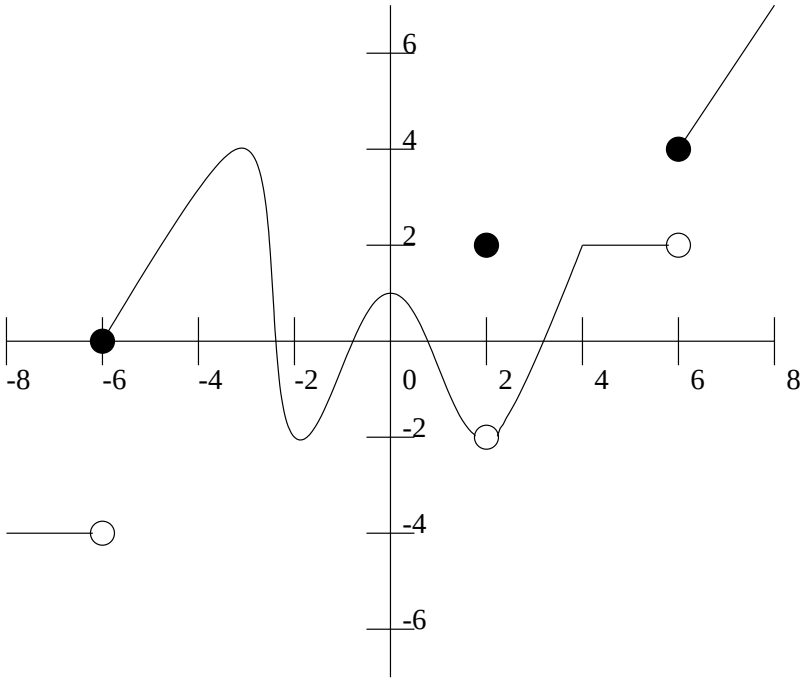
3. (*15 points*) Compute the following. Do not simplify your answers.

(a) $\displaystyle\int \left(\sqrt[5]{x} + x^{-1} + \frac{1}{1+x^2} \right) \, dx$

(b) $\displaystyle\int_0^{\pi/4} \sec^2 x \, dx$

(c) $\displaystyle\frac{d}{dx} \int_1^x e^{-t^2} \, dt$

4. (*15 points*) Use the graph of $f(x)$ pictured here to answer the questions below. *(You do not need to show your work or justify your answers, just fill in the blanks provided.)*



Do not worry about the behaviour at the edges of the graph; that is at $x = -8$ and $x = 8$.

(a) $\lim_{x \to 2} f(x) =$ _____

(b) $f(2) =$ _____

(c) $\lim_{x \to -2} f(x) =$ _____

(d) $\lim_{x \to 6-} f(x) =$ _____

(e) $\lim_{x \to 6+} f(x) =$ _____

(f) $\lim_{x \to 6} f(x) =$ _____

(g) $\lim_{x \to -6-} f(x) =$ _____

(h) $\lim_{x \to -6+} f(x) =$ _____

(i) $f(x)$ is not continuous at $x =$ _____

(j) $f(x)$ is not differentiable at $x =$ _____

5. (*20 points*) Suppose that the function f is defined by

$$f(x) = \begin{cases} 2x^2 \sin^2\left(\dfrac{e^x - 5}{x^3}\right), & \text{if } x \neq 0. \\ 0, & \text{if } x = 0. \end{cases}$$

(a) Is f continuous at $x = 0$? You must justify your answer.

(b) Is f differentiable at $x = 0$? If so, find $f'(0)$. You must justify your answer.

6. (*15 points*) Consider the function f defined by

$$f(x) = \frac{1}{x^2 + 1}.$$

Using the definition of derivative directly, find the equation of the tangent line to the curve $y = f(x)$ at the point $x = 0$. *(Note that absolutely no credit will be awarded for using any "shortcuts" for differentiation.)*

7. (*8 points*) Consider the function f defined by

$$f(x) = \frac{3(x-a)^2}{x(x-b)(x-c)},$$

where a, b, and c are all **different** nonzero positive constants. *(You do not need to show your work or justify your answers, just fill in the blanks provided.)*

(a) _____ What is the domain of f?

(b) _____ List any horizontal asymptotes of f.

(c) _____ List any roots of f.

For part **(d)** below, suppose that $b = a$, i.e., that f is the function

$$f(x) = \frac{3(x-a)^2}{x(x-a)(x-c)}.$$

(d) _____ List any vertical asymptotes of f.

8. (*12 points*) Decide whether the following statements are True (**T**) or False (**F**). *(You do not need to show your work or justify your answers, just circle your choice.)*

(a) **T** **F** If $f''(a) > 0$, then $f(x)$ has a local minimum at $x = a$.

(b) **T** **F** If $f''(a) = 0$, then $f(x)$ has an inflection point at $x = a$.

(c) **T** **F** If $f'(x)$ is differentiable and $f'(x)$ has a local minimum at $x = a$, then $f(x)$ has an inflection point at $x = a$.

(d) **T** **F** If $f(x)$ is continuous at $x = a$, then $f'(a)$ exists.

(e) **T** **F** If $x = x(t)$, then $\dfrac{d}{dt}(x^3) = 3x^2$.

(f) **T** **F** $\displaystyle\int_0^1 3x^2\,dx = x^3 + C.$

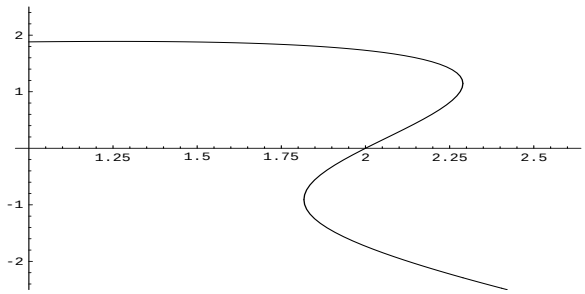
9. (*15 points*) Consider the curve given by the equation

$$x^3 + \frac{4}{3}y^3 - x^2y = 8.$$

(a) Find $\dfrac{dy}{dx}$ by implicit differentiation.

(b) Find the equation of the tangent line to the curve at the point $\left(2, \sqrt{3}\right)$.

(c) Sketch this tangent line on the graph of $x^3 + \frac{4}{3}y^3 - x^2y = 8$ below.



10. (*15 points*) Let f be a function whose **derivative** is given by

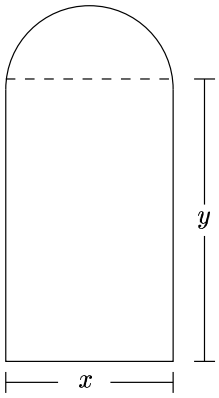
$$f'(x) = e^{-2x}x^2 - 6e^{-2x}.$$

(a) What are the critical points of $f(x)$?

(b) For each critical point of $f(x)$ you found above, use the second derivative test to determine whether it is a local maximum, local minimum, or neither.

(c) What are the inflection points of $f(x)$? Justify your answer.

11. (*15 points*) A Norman window has the shape of a rectangle surmounted by a semicircle, as illustrated to the right. If the perimeter of the entire window is $(4+\pi)$ m, find x, the width of the window, that gives the greatest area. Be sure to justify why your answer gives a maximum area.



12. (*10 points*) Suppose that a large spherical snowball is melting so that its volume is decreasing at a rate of 1 cm^3/min. Find the rate at which the radius is decreasing when the radius is 5 cm.

13. (*10 points*) Use an appropriate linear approximation to estimate the value of $e^{0.1}$.

14. (*20 points*) Consider $\displaystyle\int_0^4 \frac{1}{1+a^x}\,dx$ where $a \geq 1$ is a constant.

(a) Use a left hand Riemann sum with 4 subintervals to approximate the value of this integral. *(Naturally, your answer will involve the constant a.)*

(b) Use a right hand Riemann sum with 4 subintervals to approximate the value of this integral. *(Naturally, your answer will involve the constant a.)*

(c) How do the left hand and right hand Riemann sums compare to the true value of $\displaystyle\int_0^4 \frac{1}{1+a^x}\,dx$? Why?