Math 111 Prelim #2 - July 21, 2003

This exam has 7 problems and 2 pages.

You have 75 minutes to complete this exam. You may use a calculator, but no other aids are allowed. In order to receive full credit, you must justify your solutions. No credit will be awarded for having only the final answer, whether or not it is correct.

1. (*20 points*) For each of the following, find $f'(x)$.

(a) $f(x) = \sin(e^x - 1) - \sqrt{x^2 + 1}$

(b) $f(x) = \ln(\sqrt{x^x})$

(c) $f(x) = e^\pi - \pi^e$

(d) $f(x) = \dfrac{\tan x}{2^x}$

2. (*15 points*) A curve is given parametrically by the following equations:

$$x(t) = 4\cos^3 t, \qquad y(t) = 4\sin^3 t, \qquad 0 \le t \le 2\pi.$$

(a) Find formulas for $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$.

(b) Find $\dfrac{dy}{dx}$ when $t = \dfrac{3\pi}{4}$.

(c) Find the (x, y)-coordinates for each point on the curve where $\dfrac{dy}{dx}$ does not exist.

3. (*15 points*)

At noon the *Titanic* is 100 km west of an iceberg. The *Titanic* is sailing south at 35 km/h and the iceberg is floating north at 25 km/h. How fast is the distance between the *Titanic* and the iceberg changing at 4:00 p.m.?

4. (*10 points*) Consider the curve $x^2y^2 - 6y + 2 = 0$. Find the equation of the tangent line to the curve at the point $(2, 1)$.

5. (*10 points*) Consider the function $f(x) = \sqrt[3]{x}$. Using a linear approximation to $f(x)$ at an appropriate point, estimate the value of $\sqrt[3]{28}$.

6. (*20 points*) Suppose that $f(x) = e^x\,(x^2 + 2x + 1)$. It is known that

$$f'(x) = e^x\,(x+3)\,(x+1)$$

and

$$f''(x) = e^x\,(x+3-\sqrt{2})\,(x+3+\sqrt{2}).$$

(a) Determine the intervals on which f is increasing and the intervals on which f is decreasing.

(b) Give the x-coordinate(s) of any points at which f has a local maximum, and give the x-coordinate(s) of any points at which f has a local minimum.

(c) Determine on which intervals f is concave up and on which intervals f is concave down.

(d) Give the x-coordinate(s) of any inflection points of f.

7. (*10 points*) On Prelim # 1, you considered the function

$$f(x) = \begin{cases} 3x^2\cos^2(1/x), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0, \end{cases}$$

and showed that f is continuous at 0. Is f also differentiable at 0? Prove or disprove.