Math 111 Prelim #1 Solutions (Corrected)- July 7, 2003

3. By definition, the slope of the tangent line is $f'(3)$. Hence,

$$\begin{aligned}
f'(3) = \lim_{x\to 3} \frac{f(x)-f(3)}{x-3} &= \lim_{x\to 3} \frac{\frac{1}{\sqrt{x+1}} - \frac{1}{\sqrt{3+1}}}{x-3} \\
&= \lim_{x\to 3} \frac{2-\sqrt{x+1}}{2\sqrt{x+1}(x-3)} \\
&= \lim_{x\to 3} \frac{2-\sqrt{x+1}}{2\sqrt{x+1}(x-3)} \cdot \frac{2+\sqrt{x+1}}{2+\sqrt{x+1}} \\
&= \lim_{x\to 3} \frac{4-(x+1)}{2\sqrt{x+1}(x-3)(2+\sqrt{x+1})} \\
&= \lim_{x\to 3} \frac{3-x}{2\sqrt{x+1}(x-3)(2+\sqrt{x+1})} \\
&= \lim_{x\to 3} \frac{-1}{2\sqrt{x+1}(2+\sqrt{x+1})} \\
&= -\frac{1}{16}
\end{aligned}$$

Therefore, the equation of the tangent line is $y - f(3) = f'(3)(x-3)$, or

$$y - \frac{1}{2} = -\frac{1}{16}(x-3).$$