Prelim 1 Review

These are some of the main ideas in Chapters 2 and 3. You should also review the main ideas in Chapter 1.

1. What does it mean for a limit to exist from the right? from the left?

2. State the definition of continuity at a point. In other words, what does it mean for the function $f(x)$ to be continuous at $x = a$?

3. State the definition of derivative at a point. In other words, what is the defintion of the derivative of the function $f(x)$ at the point $x = a$?

4. State an alternative defintion for $f'(a)$.

5. State the definition of the derivative in general. That is, what is the defintion of the derivative $f'(x)$ of $f(x)$.

6. What is the relationship between differentiability and continuity? Does one guarantee the other? Are they equivalent?

7. If $f'(a) = 0$, what can you say about $f(a)$? Be precise.

8. If $f'(a) > 0$, what can you say about f near a?

9. If $f'(a) < 0$, what can you say about f near a?

10. If $f''(a) > 0$, what can you say about f near a?

11. If $f''(a) < 0$, what can you say about f near a?

12. Carefully state the definitions of horizontal asymptote and vertical asymptotes. How do you find vertical asymptotes?

13. State the intermediate value theorem.

14. State the squeeze theorem.

15. State the product rule, and the quotient rule.

Computational Practice

1. $\displaystyle\lim_{x\to 4}\frac{x^2+7x+12}{x+4}$

2. $\displaystyle\lim_{x\to -4}\frac{x^2+7x+12}{x+4}$

3. $\displaystyle\lim_{x\to\infty}\ln x$

4. $\displaystyle\lim_{x\to\infty}\tan^{-1}x=\lim_{x\to\infty}\arctan x$

5. $\displaystyle\lim_{x\to 0^+}\ln x$

6. $\displaystyle\lim_{x\to\infty}\frac{(2x+1)^3}{3x^3+1}$

7. $\displaystyle\lim_{x\to -\infty}\frac{x+1}{\sqrt{x^2+1}}$

8. $\displaystyle\lim_{x\to 2}|x-2|$

9. $\displaystyle\lim_{x\to 2}\frac{|x-2|}{x-2}$

10. $\displaystyle\lim_{x\to -2}\frac{|x-2|}{x-2}$

11. $\displaystyle\lim_{h\to 0}\frac{e^h-1}{h}$

12. $\displaystyle\lim_{h\to 0}\frac{(2+h)^{10}-1024}{h}$

13. $\displaystyle\lim_{x\to a}\frac{\sqrt[3]{x}-\sqrt[3]{a}}{x-a}$

14. Compute $f'(x)$ if

(a) $f(x) = 7x^2 - \dfrac{1}{x^2}$

(b) $f(x) = 456$

(c) $f(x) = (x - e^x)\sqrt{x}$

(d) $f(x) = \dfrac{e^x \sqrt{x}}{x+1}$

(e) $f(x) = e^x + x^e$

(f) $f(x) = e^\pi + \pi^e$

15. Suppose that $f(x) = \dfrac{x^2 + 1}{x^2 - 1}$. Without using a calculator, complete the following:

(a) What is the domain of f?

(b) What is the range of f?

(c) What are the roots of f?

(d) Compute all asymptotes of f?

(e) Find $f'(x)$ and $f''(x)$. This can be done with just the quotient rule.

(f) Carefully sketch a graph of f making sure that you include all important features of the graph.