Math 111.17 Fall 2002
Assignment #11 Solutions

3. (a) Let $f(x) = x^3 + 3x - 2k$. Then Newton's method tells us

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

Since $f'(x) = 3x^2 + 3$, substituting yields

$$\begin{aligned} x_{n+1} &= x_n - \frac{(x_n^3 + 3x_n - 2k)}{(3x_n^2 + 3)} \\ &= \frac{(3x_n^2 + 3)x_n - (x_n^3 + 3x_n - 2k)}{(3x_n^2 + 3)} \\ &= \frac{2}{3} \cdot \frac{x_n^3 + k}{x_n^2 + 1} \end{aligned}$$

(b) Use the above formula with $k = 1$, $x_0 = 1$, to conclude

$$\begin{aligned} x_0 &= 1 \\ x_1 &= \frac{2}{3} \cdot \frac{1^3 + 1}{1^2 + 1} = \frac{2}{3} \approx 0.66667 \\ x_2 &\approx 0.59829 \\ x_3 &\approx 0.59607 \\ x_4 &\approx 0.59607 \end{aligned}$$

$\therefore$ Accurate to 5 decimal places, $x^3 + 3x - 2 = 0$ has a root at 0.59607.

4. (a) Notice that $f(0)$ is not defined. However, $f(1) = -2 < 0$, $f(e) = e - 2 > 0$, and f is continuous on $[1, e]$. Thus, by the Intermediate Value Theorem, f has a root in $(1, e)$ [and therefore has a root in $(0, e)$].

(b) If $f(x) = x \ln x - 2$, then $f'(x) = \ln x + 1$. Hence, Newton's Method tells us that

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n \ln x_n - 2}{\ln x_n + 1}.$$

Thus, $x_0 = 2$, $x_1 \approx 2.362464$, $x_2 \approx 2.345783$, $x_3 \approx 2.345751$, $x_4 \approx 2.345751$.

Accurate to six decimal places f has a root of 2.345751.

(c) Since $f''(x) = 1/x$, if x is near 2 [in fact, if $x > 0$], then $f''(x) > 0$ so that f is concave up. Thus, all tangent lines lie under the graph of f. This implies that in the Newton's Method scheme, all approximations will be bigger than the actual solution.