Math 105 Prelim #3 - Practice Problems

Listed below is a *selection* of problems that will provide a useful supplement to your studying. They are not meant to be either a substitute for studying, or a guaranteed means for achieving an A, but rather an important *component* in your overall preparation and review.

Solutions will be made available shortly.

1. Section 8.4 #42

2. Chapter 8 review #32

3. Chapter 8 review #46

4. Chapter 9 review #28

5. Chapter 9 review #30

6. The intelligence quotient (IQ) score, as measured by the Stanford–Binet IQ test, is normally distributed in a certain population of children. The mean IQ score is 100, and the standard deviation is 16 points. What percentage of children in the population have IQ scores

(a) of 80 or less?

(b) between 80 and 120?

(c) of 140 or more?

(d) Suppose that five children are chosen at random from the population. What is the probability that one of them will have an IQ score of 80 or less and four will have IQ scores higher than 80?

7. Suppose that a certain population of observations is normally distributed. Find the value of z^* such that 95.00% of the observations in the population are between $-z^*$ and z^* on the Z-scale.

8. For each of the following, describe a situation in which they would be the calculation of the probability out an outcome:

(a) $\left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^4 \binom{7}{3}$

(b) $\left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^4$

(c) $\frac{\binom{4}{2}\binom{5}{3}}{\binom{9}{5}}$

9. You have two fair six-sided dice; the first die has two sides numbered 2 and four sides numbered 3, and the second die has three sides numbered 1, two sides numbered 2, and one side numbered 3.

Let X be a random variable representing the sum of the two upmost faces on one roll of both dice. Note that the possible values of X are $3, 4, 5, 6$.

(a) What is $P(X = 4)$, the probability of rolling a sum of 4?

(b) What is $E(X)$, the expected value of the sum of the dice?

Suppose you roll the second die 72 times. Let Y be a random variable representing the total number of 2s observed on the upmost face. Note that the possible values of Y are $0, 1, 2, \ldots, 71, 72$.

(c) What is $E(Y)$, the expected number of 2s?

(d) Using the normal curve, what is $P(30 < Y < 40)$, the probability of rolling between 30 and 40 2s on these 72 rolls?

10. (adapted from #21 in 9.2) Bright Idea Lighting tests their light bulbs, and finds that they have a mean life of 262 hours, with a standard deviation of 41 hours. They test a sample of light bulbs of their rival, The Electric Company, and get that they last 340, 190, 150, 280, 250, 180, 380, 300, 250, and 230 hours.

(a) Find the median, mean, and standard deviation of the life of The Electric Company's light bulbs.

(b) Which brands' light bulbs have the higher mean life?

(c) Assuming the distribution of bulb life of both companies follows a normal distribution, how likely is each company to produce a light bulb that lasts 350 hours?

11. In the theme song to the 1960's television show "Secret Agent Man", we're told, "Odds are he won't live to see tomorrow." Suppose that on any given day that he's alive, the probability that the secret agent lives to see tomorrow are 49%.

(a) Today is Thursday. What's the probability that he'll still be alive next Wednesday?

(b) Suppose that there is a team of 12 secret agents. If each lives or dies independently of the others, how many do you expect to be around on Saturday?

12.

(a) Suppose your data is normally distributed with a mean of 0 and a standard deviation of 1. Within what range does the middle 90% of your data lie?

(b) Suppose the mean is 10 and the standard deviation is 1. Within what range does the middle 90% of your data lie?

13. Consider the following experiment: You roll two standard six-sided dice, and then sum the outcomes. If you repeat this experiment nine times, what is the probability that fewer than nine of the trials result in a number greater than or equal to ten?

14. It is a known fact that 70 percent of all existentialists carry a copy of Jean-Paul Sartre's *Being and Nothingness*. If five existentialists are chosen at random, what is the probability that fewer than 3 of them have a copy of this book?

15. A coin is flipped ten times. For each of the following pairs, say whether $p < q$, $p > q$, or $p = q$:

(i) $p = P(3 \text{ heads})$, $q = P(4 \text{ tails})$

(ii) $p = P(> 3 \text{ heads})$, $q = P(< 2 \text{ tails})$

(iii) $p = P(n \text{ heads})$, $q = P(10 - n \text{ tails})$, where $0 \leq n \leq 10$

16. Suppose you have a normal distribution N with mean μ and standard deviation σ. Let P_N denote probability in this distribution (not in the standard normal distribution). Find the following:

(i) $P_N(x < \mu)$

(ii) $P_N(\mu - \sigma < x < \mu + \sigma)$

(iii) $P_N(\mu - \frac{\sigma}{2} < x < \mu + \frac{\sigma}{2})$

(iv) $P_N(x > \mu + 2\sigma)$

17. **(Open-Ended)** In a famous experiment designed to test monkey intelligence, Prof. John Frink performed the following experiment on 1000 monkeys. He gave each monkey a red ball, and allowed the monkey to play with it for one minute. He then placed the same red ball along with a blue ball and a yellow ball into a box, and instructed the monkey to select the red ball. Prof. Frink recorded the ball chosen by the monkey. His data are summarized below:

ball selected	# of monkies
red	435
blue	258
yellow	311

Based on the results of his experiment, do you believe that there is adequate evidence for Prof. Frink to conclude that the monkies are able to correctly distinguish the red ball, and therefore exhibit intelligence?

(Note: In order to receive full credit, you must use techniques discussed in chapters 8 and 9 of the textbook to support your answer.)