**Math 105 Prelim #2 - Practice Problems**

Listed below is a *selection* of problems that will provide a useful supplement to your studying. They are not meant to be either a substitute for studying, or a guaranteed means for achieving an A, but rather an important *component* in your overall preparation and review.

**0.** State the definition of *conditional probability* and state *Bayes' Theorem.*

**1.** A food science class at Cornell makes a new flavor of ice cream by adding extra ingredients to vanilla ice cream. This year, the possible ingredients are: chocolate chips, peanuts, marshmallows, walnuts, cherries, and caramel.

**(a)** How many flavors can they make using exactly two additional ingredients?

**(b)** How many flavors are possible using any number of additional ingredients (including plain vanilla ice cream)?

**(c)** Suppose we make a pint of all of the possibilites in **(b)**. What is the probability that it has no nuts in it?

**2.** One out of five students eating at Trillium order french fries, and one out of four get ketchup. Three out of ten get either french fries or ketchup.

**(a)** What is the probability that a student who orders fries gets ketchup?

**(b)** What is the probability that a student who gets ketchup does not order fries?

**3.** Two fair dice are rolled.

**(a)** Find the probability that the sum is no more than 6.

**(b)** Find the probability that the sum is at least 9 given that at least one of the die shows a 5.

**4.** Two hands of 5 cards are drawn from a standard deck of 52 cards.

**(a)** What is the probability that the first hand contains 3 cards of the same values and 2 cards of other different values? (This hand is known as "three-of-a-kind.")

**(b)** What is the probability that the first hand has 5 clubs and the second hand 5 hearts? (These two hands are known as a "club flush" and a "heart flush," respectively.)

**5.** According to the *National Lightning Safety Institute* the chance of being struck by lightning is roughly 1/280 000. (http://www.lightningsafety.com/nlsi_pls/probability.html)

The card game of bridge is played with a standard deck of 52 cards by 4 players. Each player is dealt 13 cards. A perfect bridge hand occurs when a player is dealt all 13 cards of a single suit. Suppose that the deck is well shuffled, and the top 13 cards are dealt to the first player.

To play the New York "Lotto" you select 6 different numbers from among $\{1, 2, 3, \ldots, 59\}$. Each week 6 numbers are drawn at random without replacement by the Lottery Corporation. In order to win the grand prize a player's 6 numbers must match the 6 numbers drawn that week. (The order the winning numbers are drawn does not matter.)

Pat is a compulsive gambler and plays both birdge and "Lotto" each week. Consider the events (a) as first player, Pat will be dealt a perfect hand at bridge; (b) Pat will win the "Lotto"; and (c) Pat will be struck by lightning.

Which of these three events is most likely to occur this week? Why?

**6.** To survive the pressures of university, many students resort to consuming caffeine. Common sources of caffeine are coffee, tea, and cola.

A survey was conducted among Cornell students and that it was found that 55% of them drink cola, 45% of them drink coffee, and 25% of them drink tea.

It was also found that 15% drink both tea and cola, 5% drink all three beverages, 25% drink both cola and coffee, and 5% drink only tea.

Suppose that a Cornell student is selected at random.

**(a)** What is the probability that the student drinks only coffee?

**(b)** What is the probability that the student drinks none of these beverages?

**7.** According to Dr. Jim Mass in Psychology 101, 43% of adults are moderately to severely sleep deprived, and "college students are walking zombies." It is has been found that college students require at least 9 hours sleep, and that caffeine causes fragmented sleep.

In the same survey of Cornell students as in the previous problem, it was found that 50% of them sleep for less than 6 hours each night, 35% of them sleep between 7 and 9 hours each night, and only 15% of them sleep for over 9 hours each night.

**(a)** Suppose that a Cornell student is chosen at random, and that the student sleeps for more than 7 hours each night. What is the probability that the student actually sleeps for more than 9 hours each night?

**(b)** Suppose that 5 Cornell students are chosen at random. What is the probability that exactly 2 of them receive the required sleep, namely over 9 hours each night?

**8.** Suppose you roll a standard six-sided die twice. Let $E$ be the event, "the first roll is a 5," and let $F$ be the event, "the sum of the two rolls is greater than 9." Are $E$ and $F$ independent? Why or why not?

**9.** A hungry graduate student has decided to steal food from one of his classmates' desks. He chooses randomly between the desks of Adam, Beatrix, and Charlie, and then chooses at random one piece of food from the chosen desk. There are 2 bagels and 3 muffins on Adam's desk, 3 bagels and 1 muffin on Beatrix's desk, and 10 muffins on Charlie's desk.

**(a)** Draw a tree diagram that represents the graduate student's thievery.

**(b)** Given that the stolen piece of food was a muffin, use Bayes' Theorem to determine the probability that it came from Charlie's desk.

**10.** In how many ways can three people sit in a row of five chairs? In how many ways can this be done so that no two people are sitting in adjacent seats?

**11.** Suppose a student is taking a 4-question multiple choice quiz where each question has three possible answers ($A$, $B$, or $C$).

**(a)** If the student guesses randomly on every question, what is the probability she gets a perfect score on the quiz?

**(b)** What is the probability she does not get a grade of zero?

**12.** Out of a crate of 10 apples, two are rotten. Suppose you choose three apples at random. What is the probability that exactly one of the apples in your choice is rotten?

**13.**

**(a)** A group of 10 people wants to start a radio station. How many ways can they pick a station manager, a program manager, and 3 music directors?

**(b)** Evelyn and Frank want to help start the station, but are lazy. If they are two of the ten people above, what is the probability that neither is (randomly) selected for one of the positions?

**14.** A bag contains 12 oranges 3 of which are rotten. How many samples of 4 oranges from this bag contain less than 2 rotten oranges?

**15.** Ask Marilyn!

**(a)** Page 343 #32

**(b)** Page 359 #19

**(c)** Page 374 #51

**(d)** Page 407 #25

**16.**

**(a)** Page 335 #51

**(b)** Page 347 #70 ab, #71 ab

**(c)** Page 363 #63

**(d)** Page 398 #35, #47

# Brief Solutions

Warning: Errors are possible!

Note that $C(n,r) = \binom{n}{r}$.

**Problem 0:**
Since $P(E \cap F) = P(E|F)P(F)$, we have

$$P(E|F) = \frac{P(E \cap F)}{P(F)}.$$

Note that $P(E \cap F) = P(F \cap E)$ so that

$$P(E|F)P(F) = P(F|E)P(E) \quad \text{or} \quad P(F|E) = \frac{P(E|F)P(F)}{P(E)}.$$

Since we can write $E = (E \cap F) \cup (E \cap F')$, as a disjoint union, we use the definition of conditional probability to find $P(E) = P(E \cap F) + P(E \cap F') = P(E|F)P(F) + P(E|F')P(F')$. We now substitute this into the above equation for $P(E)$ to derive Bayes' Theorem:

$$P(F|E) = \frac{P(E|F)P(F)}{P(E|F)P(F) + P(E|F')P(F')}.$$

The more general form is as follows. Suppose that $F_1$, $F_2$, $\ldots$, $F_n$ partition the sample space $S$. Then,

$$P(F_j|E) = \frac{P(E|F_j)P(F_j)}{P(E|F_1)P(F_1) + P(E|F_2)P(F_2) + \cdots + P(E|F_n)P(F_n)}.$$

**Problem 1:**
**(a)** $C(6,2)$ **(b)** $2^6$ (or $C(6,0) + \cdots + C(6,6)$) **(c)** $2^4/2^6 = 1/4$

**Problem 2:**
**(a)** $P(f) = 1/5$, $P(k) = 1/4$, $P(f \cup k) = 3/10$. So $P(f \cap k) = 1/5 + 1/4 - 3/10 = 3/20$. Then $P(k|f) = (3/20)/(1/5) = 3/4$ **(b)** $P(f'|k) = 1 - P(f|k) = 1 - (3/20)/(1/4) = 1 - 3/5 = 2/5$.

**Problem 3:**
**(a)** $15/36$ **(b)** $1/2$

**Problem 4:**
**(a)** $C(13,1)C(4,3)C(12,2)C(4,1)C(4,1)/C(52,5)$ **(b)** $C(13,5)C(13,5)/(C(52,5)C(47,5))$

**Problem 5:**
$P(\text{Win "Lotto"}) = 1/C(59,6) = 1/45\,057\,474$ and $P(\text{perfect bridge hand}) = 4{\times}13!/P(52,13) = 1/C(51,12) \approx 1/158\ 700\ 000\ 000$ so it is most likely that Pat will be struck by lightning (about 16 times more likely than winning the lottery and about 500 000 times more likely than being dealt a perfect hand).

**Problem 6:**
**(a)** 0.15 **(b)** 0.25

**Problem 7:**
**(a)** $15/(15+35) = 30\%$ **(b)** $\approx .138$

**Problem 8:**
No, $P(E)P(F) = 1/36$ does not equal $P(E \cap F) = 2/36$.

**Problem 9:**
**(a)** Draw de tree. The first set of branches should be $A$, $B$, $C$, each with weight 1/3. Each of those letters should have a muffin branch and a bagel branch. For $A$, the muffin branch has weight 0.6 and bagel has 0.4; for $B$ muffin has 0.25 and bagel has 0.75; for $C$ muffin has 1, and bagel has 0. **(b)** 20/37

**Problem 10:**
There are 60 ways for three people in five chairs, and there are 6 ways for no two adjacent.

**Problem 11:**
**(a)** 1/81 **(b)** 65/81

**Problem 12:**
7/15

**Problem 13:**
**(a)** 5040 **(b)** 2/9

**Problem 14:**
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