Math 105 Prelim #1 - Practice Problems

Listed below is a *selection* of problems that will provide a useful supplement to your studying. They are not meant to be either a substitute for studying, or a guaranteed means for achieving an A, but rather an important *component* in your overall preparation and review.

Standard Problems

1. Bob and Mary are selling cupcakes for 50 cents and muffins for one dollar at a bake sale. Bob sells 20 cupcakes and 24 muffins, and Mary sells 30 cupcakes and 12 muffins. It took them each a half-hour to bake every batch of 10 cupcakes, and one hour to bake each batch of 12 muffins. Use matrix multiplication to determine

(a) how much money Bob and Mary each made at the bake sale, and

(b) how long it took each of them to bake the goods they sold.

2. Let U be the set of months: {January, February, March, April, May, June, July, August, September, October, November, December}. Let A be the set of months beginning with a consonant, B be the odd numbered months, and C be the first six months of the year. Describe the set $(A \cup B') \cap C$ in words, and list all elements of the set.

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7. Find the inverse of:

$$A = \begin{bmatrix} 2 & 0 \\ 3 & -1 \end{bmatrix}.$$

8. On the last sunny day in Ithaca this year, eight friends went on a hike. Everyone brought water or juice to drink. Seven of them brought water, and five brought juice. How many brought both water and juice? (You may find it helpful to include an appropriate Venn diagram.)

9. Find the inverse of

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 3 & 8 \\ 4 & 8 & 23 \end{bmatrix}.$$

10. Find, if possible, the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}.$$

11. Find a matrix M so that

$$\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} \cdot M = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

12. Joe has 20 coins, each of which is either a dime or a nickel. His coins are worth a total of \$1.65. Determine how many of each type of coin he has.

13. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$.

(a) Find A^{-1}.

(b) Use A^{-1} to solve the matrix equation $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$.

14. Of 100 surveyed television viewers, it was found that 30 men watched a certain sitcom, while 35 women did not. If a total of 40 people watched the sitcom, how many men did *not* watch it?

15. True or False.

(a) $P(A \cup B) = P(A) + P(B)$ for any events A and B.

(b) Any system of 2 equations in 2 variables must have a unique solution, since we can just isolate the second variable and substitute it back to find the first variable.

(c) For any sets A and B,

$$(A \cap B') \cup (B \cap A') = (A \cup B) \cap (A \cap B)'.$$

(Hint: Draw a Venn diagram.)

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Challenge Problems

17. Suppose Terrance plays the following boring game with himself: he flips a coin repeatedly until he flips heads, at which time he stops. Suppose also that if Terrance does not flip heads in four tries, he simply gives up.

(a) Write out the corresponding sample space for Terrance's game.

(b) What is the probability that Terrance never flips heads?

18. Consider the system of equations given by

$$\begin{cases} 3x - 2y + 3z = -2 \\ -2x + 3y - 2z = 3. \end{cases}$$

Find, in parametric form, all possible solutions of this system of equations.

19. For what values of a and b does the following system have

(a) a unique solution,

(b) no solutions, or

(c) infinitely many solutions.

$$\begin{cases} x + y + z = 3 \\ x + 2y - az = 1 \\ 2x + y + 3z = b. \end{cases}$$

20. Consider the matrix equation $AX = B$ where

$$A = \begin{bmatrix} 1 & -8 & 3 & 8 \\ 2 & -1 & 3 & 1 \\ 1 & 2 & 1 & -2 \end{bmatrix}, \quad X = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}, \quad \text{and } B = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

Find a condition on a, b, and c so that $AX = B$ is consistent.

21. As you know, the formula for the least squares line through the data points $(x_1, y_1), \ldots (x_n, y_n)$ is given by $Y = mx + b$, where m and b satisfy the systems of equations

$$\begin{cases} nb + (\sum x)m = \sum y \\ (\sum x)b + (\sum x^2)m = \sum (xy). \end{cases}$$

Solve this system for m and b.

Brief Solutions

Warning: Errors are possible!

Problem 1:
(a) Bob earned \$34, Mary earned \$27 **(b)** Bob baked for 3 hours, Mary for 2 1/2 hours

Problem 2:
$(A \cup B') \cap C = \{\text{January, February, March, April, May, June}\}$

Problems 3, 4, 5, 6:
See back of text, or student solutions manual.

Problem 7:
Do $.5R_1 \mapsto R_1$ and $3R_1 - R_2 \mapsto R_2$ to produce

$$A^{-1} = \begin{bmatrix} .5 & 0 \\ 1.5 & -1 \end{bmatrix}.$$

Problem 8:
Since $n(W \cup J) = n(W) + n(J) - n(W \cap J)$ we have that $8 = 7 + 5 - n(W \cap J)$; thus $n(W \cap J) = 4$.

Problem 9:

$$\begin{bmatrix} 5 & -7 & 2 \\ 9 & -8 & 2 \\ -4 & 4 & -1 \end{bmatrix}$$

Problem 10:
There is no inverse.

Problem 11:

$$M = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Problem 12:
13 dimes and 7 nickels

Problem 13:
(a)

$$A^{-1} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \end{bmatrix}$$

Problem 14:
25

Problem 15:
(a) False **(b)** False **(c)** True

Problem 16:
See back of text, or student solutions manual.

Problem 17:
(a) $S = \{H, TH, TTH, TTTH, TTTT\}$. Note that each of these outcomes is *not* equally likely. In fact, $P(H) = 1/2$, $P(TH) = 1/4$, $P(TTH) = 1/8$, $P(TTTH) = 1/16$, and $P(TTTT) = 1/16$. Draw a tree diagram to see this. **(b)** 1/16

Problem 18:
$(-z, 1, z)$

Problem 19:
(a) $a \neq 0$ **(b)** $a = 0$ and $b \neq 8$ **(c)** $a = 0$ and $b = 8$

Problem 20:
$a - 2b + 3c = 0$

Problem 21:

$$m = \frac{n \sum(xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} \quad \text{and} \quad b = \frac{\sum y - m(\sum x)}{n}$$