Math 105 Fall 2003 Solutions Assignment 8 [100pts] 8.4 #50 [10 pts] There are  $\binom{6}{3}$  ways to select the three mice that will recover. Each way occurs with probability  $(0.70)^3(0.30)^3$ . Thus,

 $P(\text{exactly 3 of 6 mice recover}) = \binom{6}{3} (0.70)^3 (0.30)^3 \approx .185.$ 

8.4 #56 [5 pts each] (a) There are  $\binom{83}{10}$  ways to select the ten inoculated people that will get the flu. Each way occurs with probability  $(0.20)^{10}(0.80)^{73}$ . Thus,

 $P(\text{exactly 10 people inoculated get the flu}) = \binom{83}{10} (0.20)^{10} (0.80)^{73} \approx .0210.$ 

(b) Note that  $P\{\text{no more than } 4\} = P\{\text{exactly } 0\} + P\{\text{exactly } 1\} + P\{\text{exactly } 2\} + P\{\text{exactly } 4\}.$  Therefore,

P(no more than 4 people inoculated get the flu) =

$$\binom{83}{0} (0.20)^0 (0.80)^{83} + \binom{83}{1} (0.20)^1 (0.80)^{82} + \binom{83}{2} (0.20)^2 (0.80)^{81} + \binom{83}{3} (0.20)^3 (0.80)^{80} + \binom{83}{4} (0.20)^4 (0.80)^{79} \approx 8.004 \times 10^{-5}.$$

(c) There is only one way (i.e.,  $\binom{83}{0}$  ways) in which no one gets the flu. Thus,

 $P(\text{no people inoculated get the flu}) = {\binom{83}{0}} (0.20)^{83} (0.80)^0 = (0.20)^{83} \approx 9.046 \times 10^{-9}.$ 

**8.4** #64 [10 pts] Note that {fewer than 8} = {at least 8}'. Thus, P(fewer than 8) = 1 - P(at least 8) = 1 - P(exactly 8) - P(exactly 9) - P(exactly 10). Therefore,

$$P(\text{fewer than 8}) = 1 - {\binom{10}{8}} (0.2)^8 (0.8)^2 - {\binom{10}{9}} (0.2)^9 (0.8)^1 - {\binom{10}{10}} (0.2)^{10} (0.8)^0 \approx .999922.$$

**8.5** #4 [10 pts] Let X be the number of black balls drawn. Then the possible values of X are 0, 1, or 2. Thus,

$$P(X=0) = \frac{4}{6} \cdot \frac{3}{5} = \frac{6}{15}, \ P(X=1) = \frac{4}{6} \cdot \frac{2}{5} + \frac{2}{6} \cdot \frac{4}{5} = \frac{8}{15}, \ P(X=2) = \frac{2}{6} \cdot \frac{1}{5} = \frac{1}{15}.$$

**8.5** #10 [10 pts] By definition we have,

$$E(y) = y_1 P(y_1) + y_2 P(y_2) + y_3 P(y_3) + y_4 P(y_4) = 4 \times .4 + 6 \times .4 + 8 \times .05 + 10 \times .15 = 5.9.$$

**8.5** #14 [10 pts] By definition,  $E(x) = x_1P(x_1) + x_2P(x_2) + x_3P(x_3) + x_4P(x_4) + x_5P(x_5)$ . We can read the appropriate probabilities from the histogram to find that

$$E(x) = 2 \times .2 + 4 \times .3 + 6 \times .2 + 8 \times .1 + 10 \times .2 = 5.6.$$

**8.5** #22 [15 pts] Let W be the number of women selected. The possible values for W are 0, 1, or 2. We can compute the probability distribution of W as

$$P(W=0) = \frac{5}{7} \cdot \frac{4}{6} = \frac{10}{21}, \quad P(W=1) = \frac{5}{7} \cdot \frac{2}{6} + \frac{2}{7} \cdot \frac{5}{6} = \frac{10}{21}, \quad P(W=2) = \frac{2}{7} \cdot \frac{1}{6} = \frac{1}{21}.$$

Thus,

$$E(W) = 0 \times P(W = 0) + 1 \times P(W = 1) + 2 \times P(W = 2) = \frac{10}{21} + \frac{2}{21} = \frac{12}{21} \approx .5714.$$

**8.5** #34 [5 pts each] (a) Let X be the amount of damage (in millions of dollars) under seeding and let Y be the amount of damage (in millions of dollars) under not seeding. Then

$$E(X) = .038 \times 335.8 + .143 \times 191.1 + .392 \times 100 + .255 \times 46.7 + .172 \times 16.3 \approx 94.0,$$

and

$$E(Y) = .054 \times 335.8 + .206 \times 191.1 + .480 \times 100 + .206 \times 46.7 + .054 \times 16.3 \approx 116.0.$$

(b) The option to seed should therefore be chosen (since 94 < 116).

**8.5** #48 [10 pts] Let X be the expected payout (in \$) from the lottery. We easily compute E(X) to be

$$E(X) = 100\ 000 \times \frac{1}{2\ 000\ 000} + 40\ 000 \times \frac{2}{2\ 000\ 000} + 10\ 000 \times \frac{2}{2\ 000\ 000} = \frac{200\ 000}{2\ 000\ 000} = 0.10.$$

Thus, since it costs 50¢ in time, paper, and stamps to enter, and your expected winnings are only 10¢, it is **not** worth entering the lottery. (On the average, you can expect to win -40¢!)