HOMEWORK 2 SOLUTIONS

SECTION 2.2

20)

Written in matrix form, the problem becomes: $\left[\begin{array}{cc|c} 4 & -2 & 3 \\ -2 & 3 & 1 \end{array}\right]$. Row-reducing gives:

$$2R_2 + R_1 \to R_1 : \left[\begin{array}{cc|c} 0 & 4 & 5 \\ -2 & 3 & 1 \end{array}\right]$$

$$\frac{-3}{4}R_1 + R_2 \to R_2 : \left[\begin{array}{cc|c} 0 & 4 & 5 \\ -2 & 0 & \frac{-11}{4} \end{array}\right]$$

$$\frac{1}{4}R_1 \to R_1 \text{ and } \frac{-1}{2}R_2 \to R_2 : \left[\begin{array}{cc|c} 0 & 1 & \frac{5}{4} \\ 1 & 0 & \frac{11}{8} \end{array}\right]$$

$$R_1 \leftrightarrow R_2 : \left[\begin{array}{cc|c} 1 & 0 & \frac{11}{8} \\ 0 & 1 & \frac{5}{4} \end{array}\right]$$

So there is a unique solution to the system: $x = \frac{11}{8}$ and $y = \frac{5}{4}$.

32)

Again, we put the system of equations in matrix form:

$$\left[\begin{array}{ccc|c} 3 & -6 & 3 & 15 \\ 2 & 1 & -1 & 2 \\ -2 & 4 & -2 & 2 \end{array}\right]$$

$$R_2 + R_3 \to R_3 \text{ and } \frac{1}{3}R_1 \to R_1 \left[\begin{array}{ccc|c} 1 & -2 & 1 & 5 \\ 2 & 1 & -1 & 2 \\ 0 & 5 & -3 & 4 \end{array}\right]$$

$$(-2)R_1 + R_2 \to R_2 : \left[\begin{array}{ccc|c} 1 & -2 & 1 & 5 \\ 0 & 5 & -3 & -28 \\ 0 & 5 & -3 & 4 \end{array}\right]$$

$$(-1)R_2 + R_3 \to R_3 : \left[\begin{array}{ccc|c} 1 & -2 & 1 & 5 \\ 0 & 5 & -3 & -28 \\ 0 & 0 & 0 & 32 \end{array}\right].$$

We can stop now, since the last row, as an equation, means "$0 = 32$." Therefore, the system is inconsistent (i.e., it has no solutions).

44)

Denote the amounts of yarn ordered from Toronto, Montreal, and Ottawa by x, y, and z, respectively. Since the total number of units ordered was 100, we have: $x+y+z=100$. Since the total delivery cost was 5990, this gives us $80x+50y+65z=5990$. Finally, since the same amount was ordered from Toronto and Ottawa, we have $x=z$, which can be written $x-z=0$. Putting this data into a matrix, we have: $\begin{bmatrix} 1 & 1 & 1 & | & 100 \\ 80 & 50 & 65 & | & 5990 \\ 1 & 0 & -1 & | & 0 \end{bmatrix}$. Now we row-reduce:

$$(-80)R_1+R_2 \to R_2 \text{ and } (-1)R_1+R_3 \to R_3: \begin{bmatrix} 1 & 1 & 1 & | & 100 \\ 0 & -30 & -15 & | & 2010 \\ 0 & -1 & -2 & | & -100 \end{bmatrix}$$

$$\frac{-1}{30}R_2 \to R_2: \begin{bmatrix} 1 & 1 & 1 & | & 100 \\ 0 & 1 & \frac{1}{2} & | & 67 \\ 0 & -1 & -2 & | & -100 \end{bmatrix}$$

$$(-1)R_2+R_1 \to R_1 \text{ and } R_2+R_3 \to R_3: \begin{bmatrix} 1 & 0 & \frac{1}{2} & | & 33 \\ 0 & 1 & \frac{1}{2} & | & 67 \\ 0 & 0 & -\frac{3}{2} & | & -33 \end{bmatrix}$$

$$\frac{-2}{3}R_3 \to R_3: \begin{bmatrix} 1 & 0 & \frac{1}{2} & | & 33 \\ 0 & 1 & \frac{1}{2} & | & 67 \\ 0 & 0 & 1 & | & 22 \end{bmatrix}$$

$$\frac{-1}{2}R_3+R_1 \to R_1 \text{ and } \frac{-1}{2}R_3+R_2 \to R_2: \begin{bmatrix} 1 & 0 & 0 & | & 22 \\ 0 & 1 & 0 & | & 56 \\ 0 & 0 & 1 & | & 22 \end{bmatrix}.$$

Therefore, we have the unique solution $(x,y,z)=(22,56,22)$.

SECTION 2.3

4)

The size of a matrix A is always described as (number of rows of A) x (number of columns of A). Since the matrix in question has 2 rows and 4 columns, it is a 2 x 4 matrix, and so the answer is 'false.'

6)

Two matrices are equal only if they are the same size and their corresponding entries are equal. Since the two matrices are not the same size, they cannot possibly be equal, and so the answer is 'false.'

24)

Since matrix addition (and therefore subtraction) is componentwise, we have:

$$\begin{bmatrix} 1 & 3 & -2 \\ 4 & 7 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 6 & -5 \\ 0 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 1-3 & 3-6 & -2-(-5) \\ 4-0 & 7-4 & 1-2 \end{bmatrix} = \begin{bmatrix} -2 & -3 & 3 \\ 4 & 3 & -1 \end{bmatrix}.$$

30)
$$\begin{bmatrix}-1 & -1\\ -1 & 0\end{bmatrix}+\begin{bmatrix}4 & 3\\ 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 1\\ 1 & 4\end{bmatrix}=\begin{bmatrix}-1+4+1 & -1+3+1\\ -1+1+1 & 0+2+4\end{bmatrix}=\begin{bmatrix}4 & 3\\ 1 & 6\end{bmatrix}.$$

SECTION 2.4

18)
$$\begin{bmatrix}5 & 2\\ 7 & 6\\ 1 & 0\end{bmatrix}\begin{bmatrix}1 & 4 & 0\\ 2 & -1 & -2\end{bmatrix}=\begin{bmatrix}5(1)+2(2) & 5(4)+2(-1) & 5(0)+2(2)\\ 7(1)+6(2) & 7(4)+6(-1) & 7(0)+6(2)\\ 1(1)+0(2) & 1(4)+0(-1) & 1(0)+0(2)\end{bmatrix}=\begin{bmatrix}9 & 18 & 4\\ 19 & 22 & 12\\ 1 & 4 & 0\end{bmatrix}$$

20)
$$\begin{bmatrix}6 & 0 & -4\\ 1 & 2 & 5\\ 10 & -1 & 3\end{bmatrix}\begin{bmatrix}1\\ 2\\ 0\end{bmatrix}=\begin{bmatrix}6(1)+0(2)-4(0)\\ 1(1)+2(2)+5(0)\\ 10(1)-1(2)+3(0)\end{bmatrix}=\begin{bmatrix}6\\ 5\\ 8\end{bmatrix}$$

28)
$$\begin{bmatrix}4 & 3\\ 1 & 2\\ 0 & -5\end{bmatrix}\left(\begin{bmatrix}2 & -2\\ 1 & -1\end{bmatrix}\begin{bmatrix}10\\ 0\end{bmatrix}\right)=\begin{bmatrix}4 & 3\\ 1 & 2\\ 0 & -5\end{bmatrix}\begin{bmatrix}20\\ 10\end{bmatrix}=\begin{bmatrix}110\\ 40\\ -50\end{bmatrix}$$

48a)
Sales for Store I increased by 25 percent, meaning the new sales amounts are $1.25=\frac{5}{4}$ times the old amounts. For Store II, this number is $\frac{4}{3}$, and for Store III it is $\frac{11}{10}$. Since the first column in the given matrix represents Store I, and so on, the desired 3 x 1 matrix is: $\begin{bmatrix}\frac{5}{4}\\ \frac{4}{3}\\ \frac{11}{10}\end{bmatrix}$

48b)
$$\begin{bmatrix}88 & 105 & 60\\ 48 & 72 & 40\\ 16 & 21 & 0\\ 112 & 147 & 50\end{bmatrix}\begin{bmatrix}\frac{5}{4}\\ \frac{4}{3}\\ \frac{11}{10}\end{bmatrix}=\begin{bmatrix}316\\ 200\\ 48\\ 391\end{bmatrix}.$$

SECTION 2.5

6)
$\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & -2\\ 1 & -1 & 0\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\ 1 & 0 & 0\\ 0 & -1 & 0\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 1\end{bmatrix}$. Since the matrix obtained is not the identity matrix, the two matrices are not inverses of each other.

24)

We use the technique of 'attaching' the 3x3 identity matrix, and then row-reducing our original matrix:

$$\left[\begin{array}{ccc|ccc} 2 & 2 & -4 & 1 & 0 & 0 \\ 2 & 6 & 0 & 0 & 1 & 0 \\ -3 & -3 & 5 & 0 & 0 & 1 \end{array}\right]$$

$$-R_1 + R_2 \to R_2 \text{ and } 2R_3 \to R_3 : \left[\begin{array}{ccc|ccc} 2 & 2 & -4 & 1 & 0 & 0 \\ 0 & 4 & 4 & -1 & 1 & 0 \\ -6 & -6 & 10 & 0 & 0 & 2 \end{array}\right]$$

$$3R_1 + R_3 \to R_3 : \left[\begin{array}{ccc|ccc} 2 & 2 & -4 & 1 & 0 & 0 \\ 0 & 4 & 4 & -1 & 1 & 0 \\ 0 & 0 & -2 & 3 & 0 & 2 \end{array}\right]$$

$$\frac{1}{2}R_1 \to R_1, \frac{1}{4}R_2 \to R_2 \text{ and } \frac{-1}{2}R_3 \to R_3 : \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 1 & \frac{-1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 1 & \frac{-3}{2} & 0 & -1 \end{array}\right]$$

$$-R_2 + R_1 \to R_1 : \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & \frac{3}{4} & \frac{-1}{4} & 0 \\ 0 & 1 & 1 & \frac{-1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 1 & \frac{-3}{2} & 0 & -1 \end{array}\right]$$

$$-R_3 + R_2 \to R_2 \text{ and } 3R_3 + R_1 \to R_1 : \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{-15}{4} & \frac{-1}{4} & -3 \\ 0 & 1 & 0 & \frac{5}{4} & \frac{1}{4} & 1 \\ 0 & 0 & 1 & \frac{-3}{2} & 0 & -1 \end{array}\right]$$

So the inverse of the original matrix is: $\begin{bmatrix} \frac{-15}{4} & \frac{-1}{4} & -3 \\ \frac{5}{4} & \frac{1}{4} & 1 \\ \frac{-3}{2} & 0 & -1 \end{bmatrix}$.

40)

First we find the inverse of the coefficient matrix:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$R_1 + R_3 \to R_3 : \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right]$$

$$-R_3 + R_2 \to R_2 : \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right]$$

$$-R_2 + R_1 \to R_1 \text{ and } -R_2 + R_3 \to R_3 : \begin{bmatrix} 1 & 0 & 0 & | & 2 & -1 & 1 \\ 0 & 0 & 1 & | & -1 & 1 & -1 \\ 0 & 1 & 0 & | & 2 & -1 & 2 \end{bmatrix}$$

$$R_2 \leftrightarrow R_3 : \begin{bmatrix} 1 & 0 & 0 & | & 2 & -1 & 1 \\ 0 & 1 & 0 & | & 2 & -1 & 2 \\ 0 & 0 & 1 & | & -1 & 1 & -1 \end{bmatrix}$$

So the inverse of the coefficient matrix is $\begin{bmatrix} 2 & -1 & 1 \\ 2 & -1 & 2 \\ -1 & 1 & -1 \end{bmatrix}$. Since the solution of the system is given by the product of the inverse of the coefficient matrix with $\begin{bmatrix} 3 \\ 8 \\ 4 \end{bmatrix}$, we have $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & -1 & 1 \\ 2 & -1 & 2 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 8 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix}$.