Math 105 Fall 2003 Homework 10 Solutions [100 points]

Section 9.3

#12 [10 pts]. According to the normal curve table, the area to the left of $z = -1.74$ is 0.0409, and the are to the left off $z = -1.02$ is 0.1539. So the percent between the z-scores is $0.1539 - 0.0409 = 0.1130 = \boxed{11.30\%.}$

#18 [10 pts]. We want 25% to the right of z, and the values in the table are the area to the left of z. Since the total area under the curve is 1, we want $1 - 0.25 = 0.75$ to the left of z. The closest value in the table is $z = \boxed{0.67}$ which has .7486 to left, and thus .2514 = 25.14% to the right.

#24 [10 pts]. We have $\mu = 500$ and $\sigma = 100$. We want bulbs lasting between 650 and 780 hours, so we want the z-scores for 650 and 780, respectively.

$$\begin{aligned}\text{For } 650: \quad z &= \frac{650 - 500}{100} \\ &= \frac{150}{100} = 1.50 \\ \text{For } 780: \quad z &= \frac{780 - 500}{100} \\ &= \frac{280}{100} = 2.80\end{aligned}$$

The area to the left of $z = 1.50$ is 0.9332, and the area to the left of $z = 2.80$ is 0.9974, to the area between the two is $0.9974 - 0.9332 = 0.0642$, which is the probability that a light bulb will last between 650 and 780 hours. The number of light bulbs that are then expected to last between 650 and 780 hours is, by the binomial expected value, $10000 \cdot 0.0642 = \boxed{642.}$

#28 [10 pts]. Since we want the middle 80% of bulbs, we want z-scores corresponding to the areas 0.10 and 0.90. The closest area to 0.10 is 0.1003, which corresponds to a z-score of -1.28. The closest area to 0.90 is 0.8997, which corresponds to a z-score of 1.28. So, the shortest lived bulb in the middle 80% will have a z-score of -1.28, and the longest lived will have a z-score of 1.28. Plugging these values into the equation for z-scores, and, using the mean and standard deviation given above in #24, we solve for x:

$$z = -1.28 = \frac{x - 500}{100} \Longrightarrow x = 372$$

$$z = 1.28 = \frac{x - 500}{100} \Longrightarrow x = 628$$

So the shortest lived of the middle 80% will last $\boxed{\text{372 hours}}$ and the longest will last $\boxed{\text{628 hours.}}$

#42 [10 pts]. We must first find the probability that an egg is at least 2.2 oz. We have $\mu = 1.5$ and $\sigma = 0.4$. The z-score for 2.2 is thus

$$z = \frac{2.2 - 1.5}{0.4} = \frac{0.7}{0.4} = 1.75.$$

The probability that an egg will have a z-score less than 1.75, and so will weigh less than 2.2 oz., is 0.9599. The probability that an egg is extra large, i.e., weighs more than 2.2 oz, is $1 - 0.9599 = 0.0401$. The expected value for the number of extra large eggs is $60 \cdot 0.0401 = 2.46 \cong \boxed{2 \text{ eggs.}}$

Section 9.4

#6 (a) [5 pts]. Here, $p = 1/2$, and $n = 16$. So

$$\begin{aligned} P(\text{fewer than 5 tails}) &= P(0) + P(1) + P(2) + P(3) + P(4) \\ &= \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{16} \binom{16}{0} + \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{15} \binom{16}{1} \\ &\quad + \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{14} \binom{16}{2} + \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{13} \binom{16}{3} \\ &\quad + \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{12} \binom{16}{4} \\ &= 0.00002 + 0.0002 + 0.0018 + 0.0085 + 0.0278 \\ &= \boxed{0.0383} \end{aligned}$$

(b) [10 pts]. First, we have that

$$\mu = np = 16 \cdot 0.5 = 8, \text{ and}$$

$$\sigma = \sqrt{16 \cdot 0.5 \cdot 0.5} = \sqrt{4} = 2.$$

Since we want fewer than 5 tails, we want the z-scores for -0.5 and 4.5:

$$\begin{aligned} \text{For } -0.5: \quad z &= \frac{-0.5 - 8}{2} \\ &= \frac{-8.5}{2} = -4.25 \Longrightarrow \text{ Area } \cong 0 \end{aligned}$$

$$\begin{aligned} \text{For } 4.5: \quad z &= \frac{4.5 - 8}{2} \\ &= \frac{3.5}{2} = -1.75 \Longrightarrow \text{ Area } \cong 0.0401 \end{aligned}$$

So the probability of getting less than 5 tails is $0.0401 - 0 = \boxed{0.0401.}$

#22 (a) [5 pts]. Here, $n = 20$ and $p = .025$. Using the formula for binomial probability:

$$
\begin{aligned}
P(\text{at most 1 coconut injury}) &= P(0) + P(1) \\
&= (.025)^0(.975)^{20}\binom{20}{0} + (.025)^1(.975)^{19}\binom{20}{1} \\
&= 0.603 + 0.309 \\
&= \boxed{0.912}
\end{aligned}
$$

(b) [10 pts]. Here, $n = 2000$, so we have

$$\mu = 2000 \cdot 0.025 = 50, \text{ and}$$

$$\sigma = \sqrt{2000 \cdot 0.025 \cdot 0.975} = \sqrt{48.75} \cong 6.98$$

We want the probability that 0 to 70 patients were injured by coconuts, so we want the z-scores for -0.5 and 70.5:

$$
\begin{aligned}
\text{For } -0.5: \quad z &= \frac{-0.5 - 50}{6.98} \\
&= \frac{-49.5}{6.98} = -7.09 \implies \text{Area } \cong 0 \\
\text{For } 70.5: \quad z &= \frac{70.5 - 50}{6.98} \\
&= \frac{20.5}{6.98} \cong 2.94 \implies \text{Area } \cong 0.9984
\end{aligned}
$$

So the probability that no more than 70 patients wer injured by coconuts is $0.9984 - 0 = \boxed{0.9984.}$

#28 [10 pts]. Here, $n = 1400$ and $p = 0.55$, so

$$\mu = 1400 \cdot 0.55 = 770, \text{ and}$$

$$\sigma = \sqrt{1400 \cdot 0.55 \cdot 0.45} = \sqrt{346.5} \cong 18.6$$

We want the probability that at least 750 people would vote for Edison Diest, so we want the z-scores for 749.5:

$$
\begin{aligned}
z &= \frac{749.5 - 770}{18.6} \\
&= \frac{-21.5}{18.6} \cong -1.10 \implies \text{Area } \cong 0.1357
\end{aligned}
$$

This is the probability that less than 750 would vote for Diest (the given probability is to the left of, i.e. less than, the z-score), so we want the complement. So the probability that at least 750 people would vote for Edison Diest is $1 - 0.1357 = \boxed{0.8643}$

#32 [10 pts]. Here $n = 100$ and $p = 0.5$, so

$$\mu = 100 \cdot 0.5 = 50, \text{ and}$$

$$\sigma = \sqrt{100 \cdot 0.5 \cdot 0.5} = \sqrt{25} = 5$$

We want the probability that a student will get at least 60 questions correct, so we want the z-score for 59.5:

$$\begin{aligned} z &= \frac{59.5 - 50}{5} \\ &= \frac{9.5}{5} = 1.90 \implies \text{Area } \cong 0.9713 \end{aligned}$$

This is the probability that a student would get less than 60 questions correct, so we want the complement. So the probability that a student would get at least 60 correct is $1 - 0.9713 = \boxed{0.0287}$