1.1 16 [10pts] We seek the equation of the line in the form $y = mx + b$. We know that $m = -1$ (the slope) and the point $(2, 4)$ is on the line in question. Thus $4 = (-1) \times 2 + b$ This gives $b = 6$. The equation of the line is $y = -x + 6$

1.1 22 [10pts] We seek the equation of the line in the form $y = mx + b$. We know that the points $(-2, 3/4)$ and $(2/3, 5/2)$ are on the line. Thus

$$m = \frac{5/2 - 3/4}{2/3 - (-2)} = \frac{7/4}{8/3} = 21/32.$$

To find b, let us write that $(-2, 3/4)$ is on the line, that is,

$$\frac{3}{4} = \frac{21}{32} \times (-2) + b.$$

Multiplying both sides by 16 gives $12 = -21 + 16b$, that is, $b = 33/16$. The equation of the line is $y = \frac{21}{32}x + \frac{33}{16}$ (We can now check that $(2/3, 5/2)$ is indeed on this line).

1.1 38 [10pts] The lines with equations $y = mx+b$ and $y = m'x+b'$ are perpendicular if and only if $m \times m' = -1$. The diagonal of the square $(-2, 5), (4, 5), (4, -1), (-2, -1)$ are the lines D, D' passing through $(-2, 5), (4, -1)$ for D and $(4, 5), (-2, -1)$ for D' (draw a picture). The line D has slope $m = (-1 - 5)/(4 - (-2)) = -1$. The line D' has slope $m' = (-1 - 5)/(-2 - 4) = 1$. As $m \times m' = -1$, these two lines are indeed perpendicular.

1.2 20 (a) [4pts] Draw each of the two lines by finding the coordinates of two points on each of the line (e.g., give a value for q and compute p)

b) [6pts] The equilibrium quantity and price are the coordinates of the intersection point of the supply and demand lines $p = 1.4q - .6$, $p = -2q + 3.2$ you drew in question (a). Thus these values must satisfy both equations. In particular, we must have $1.4q - .6 = -2q + 3.2$. This gives $3.4q = 3.8$, that is, $q = 19/17 \approx 1.1$. From this we find the equilibrium price $p = -2 \times (19/17) + 3.2 \approx 1$.

1.2 24 (a) [10pts] We model costs by a linear function $C(x) = mx + b$. We need to find the slope m and the intercept b. We know that $C(10,000) = 547,500$ and $C(50,000) = 737,500$, that is,

$$\begin{cases} 547,500 &= m \times 10,000 + b \\ 737,500 &= m \times 50,000 + b. \end{cases}$$

Substracting the first line to the second gives $190,000 = m \times 40,000$, that is, $m = 19/4$. To find b, we report the value of m in the first (or second) line above. This gives $547,500 = (19/4) \times 10,000 + b$. Finally, we find $b = 500,000$. The cost function is $C(x) = (19/4)x + 500,000$.

(b) [5pts] The cost of producing 100,000 items is $C(100,000) = (19/4) \times 100,000 + 500,000 = 975,000$ dollars.

(c) [2pts] The marginal cost is the slope of the cost function, i.e., $19/4 = 4.75$. That is, each additional item costs \$ 4.74 to produce.

1.3 8 (a) [3pts] Plot the data. A linear fit appears reasonable.

(b) [10pts] The least square line has equation $y = mx + b$ with m, b satisfying the system (see page 31)

$$\begin{cases} \sum y &= nb + (\sum x)m \\ \sum xy &= (\sum x)b + (\sum x^2)m. \end{cases}$$

Here $n = 5$, $\sum x = 425$, $\sum y = 6920$, $\sum xy = 622,250$, $\sum x^2 = 36,375$.Thus the system giving m and b becomes

$$\begin{cases} 6920 &= 5b + 425m \\ 622,250 &= 425b + 36,375m. \end{cases}$$

Multiply the fisrt line by 85 and substract the result to the second line. This gives $34,050 = 250m$ or $m = 136.2$. Reporting this value in the first line yields $b = -10193$.

(c) [5pts] The coefficient of correlation (see page 35, you need to compute $\sum y^2 = 14555400$) is $r \approx .97$, which indicates a good fit to the data.

(d) [5pts] If the linear trend continues following the model equation $y = 136.2x - 10193$, the household debt $y = 3300$ (in dollars) will be reach for x satisfying

$$3300 = 136.2x - 10193$$

That is, $x = 54265/136.2 \approx 99$.

1.3 12 [5pts] The coefficient of correlation is $-.05$. This means there appears to be no linear trend in this data.

2.1 12 [5pts] Perform $(-5)R1 + 2R_2$ to get the system

$$\begin{cases} 2a + 9b = 3 \\ \quad - 31b = -31 \end{cases}$$

This gives $b = 1$. Reporting in the first row gives $2a + 9 = 3$, that is, $a = -3$. The solution is $(-3, 1)$ or, equivalently, $a = -3, b = 1$. One should check that these values form indeed a solution of the original system (they do!).

2.1 24 [10pts] Follow the following steps.

$$\begin{cases} 2x + y + z = 9 \\ \quad - y + 3z = 11 \\ 3x - y + z = 9 \end{cases} \qquad (R_1 + 2R_2)$$

$$\begin{cases} 2x + y + z = 9 \\ \quad - y + 3z = 11 \\ \quad 5y + z = 9 \end{cases} \qquad (3R_1 - 2R_3)$$

$$\begin{cases} 2x + y + z = 9 \\ \quad - y + 3z = 11 \\ \quad 16z = 64 \end{cases} \qquad (5R_2 + R_3)$$

$$\begin{cases} 2x + y + z = 9 \\ \quad - y + 3z = 11 \\ \quad z = 4 \end{cases} \qquad ((1/16)R_3)$$

From the last system, we easily find $z = 4, y = 1, x = 2$. Hence $(2, 1, 4)$ is the solution, i.e., $x = 2, y = 1, z = 4$. One checks this is correct by reporting the values in the original equations.